

### INTEGER ADDITION AND SUBTRACTION

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Parent (or Guardian) signature \_\_\_\_\_

## MY WORD BANK

Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 2.5.) Key mathematical vocabulary is underlined throughout the packet.

difference

integers

negative number

positive number

sum

zero pair

### MR. MORTIMER'S MAGIC

As a child, Merrimack Mortimer loved to tinker with things and he grew to become an inventor. One of his most amazing inventions helped pigs learn to fly, but that's another story.

One of his other great inventions, he called "Magic Hot and Cold Cubes." Here we will learn about them in greater detail.

1.	2.
3.	4.
5.	6.
7.	8.
9.	10.
11.	

## USING COUNTERS TO REPRESENT INTEGERS

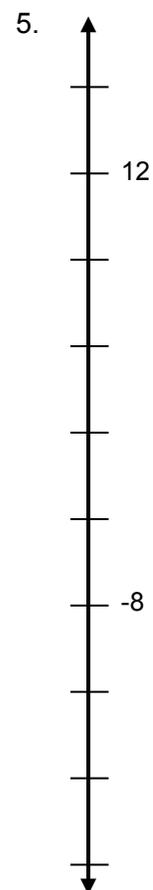
We will use counters to develop concepts about integers.

### GETTING STARTED

1. Look up integers in section 2.5 and record in My Word Bank.
2. Circle all of the numbers below that are integers.

-1      0      -5      23      1.5      -100       $\frac{1}{2}$        $-4\frac{3}{4}$

For problems 3-5, number all of the remaining tick marks.



Write =, <, or > in the space provided.

6.  $-12$  \_\_\_\_\_  $12$

7.  $-6$  \_\_\_\_\_  $-18$

8.  $25$  \_\_\_\_\_  $-50$

9.  $-125$  \_\_\_\_\_  $50$

10.  $-17$  \_\_\_\_\_  $-29$

11.  $|-5|$  \_\_\_\_\_  $|-25|$

12.  $-(-3)$  \_\_\_\_\_  $-|-3|$

13.  $-(7 - 2)$  \_\_\_\_\_  $-|7 - 2|$

### A COUNTER MODEL

We can use counters to represent positive numbers and negative numbers. Follow your teacher's directions to complete this page.

1. A positive counter has a value of 1 and looks like \_\_\_\_\_.  
 Build a value of \_\_\_\_\_ and draw it here \_\_\_\_\_.

2. A negative counter has a value of -1 and looks like \_\_\_\_\_.  
 Build a value of \_\_\_\_\_ and draw it here \_\_\_\_\_.

3. A "zero pair" has a value of 0 and looks like \_\_\_\_\_.  
 Build two zero pairs and draw them here \_\_\_\_\_. This has a value of \_\_\_\_\_.

4. Use counters to build \_\_\_\_\_ in three different ways and draw here.

a.	b.	c.
----	----	----

5. Use counters to build \_\_\_\_\_ in three different ways and draw here.

a.	b.	c.
----	----	----

	Build a value of...	Using this number of counters...	Draw here
6.			
7.			
8.			
9.			
10.			
11.			

**PRACTICE 1**

- The combination of one positive and one negative counter is called a \_\_\_\_\_.
- Describe a zero pair using Mortimer's Magic cubes.

First build each value using counters as directed. Then draw each value below.

	Build a value of...	Using ____ counters	Draw here
3.	5	the least possible number of	
4.	-6	the least possible number of	
5.	0	2	
6.	0	10	
7.	5	7	
8.	5	more than 7, but less than 11	
9.	-2	8 counters	
10.	-2	more than 2, but less than 6.	
11.	6	<i>at least 7</i>	
12.	-1	<i>more than 7</i>	

Build and draw the following situations.

<p>13. Start with a value of 4. What can you place on your work space to change this into a value of zero?</p> <p>Draw the result.</p>	<p>14. Start with a value of -3. What can you place on your work space to change this into a value of zero?</p> <p>Draw the result.</p>
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- Try to represent *any* odd value with an even number of counters. What do you notice?

## ADDING INTEGERS

We will use a counter model to generalize rules for integer addition.

### GETTING STARTED

1. The combination of four positive and four negative counters makes four \_\_\_\_\_ and has a value equal to \_\_\_\_\_.

First build each value using counters as directed. Then draw each value below.

	Build a value of...	Using _____ counters	Draw here
2.	6	the least possible number of	
3.	-4	the least possible number of	
4.	0	6	
5.	0	Between 7 and 9	
6.	4	6	
7.	4	<i>more than 7</i>	
8.	-3	<i>at least 5</i>	
9.	-3	<i>more than 7, but fewer than 11</i>	

10. Describe four zero pairs using Mortimer's magic cubes.

**ADDING INTEGERS WITH COUNTERS 1**

Use these sentence frames to help you think through integer addition. **Do not write in these.**

- Begin with a workspace that has a value equal to 0.
- Build \_\_\_\_\_  
positive/negative.
- The plus (+) means to add.
- Add \_\_\_\_\_ counter(s).  
positive/negative
- The result is \_\_\_\_\_ counter(s).  
positive/negative

Follow your teacher's directions to complete problems 1 and 2.

1.	2.
----	----

Compute each sum. Draw using positive symbols (+) and negative symbols (-).

3. $(2) + (6)$	4. $(-2) + (-6)$
5. $(-5) + (-1)$	6. $(-5) + (-5)$

7. Describe how you might add -1 and -2 using Mortimer's magic cubes. Be sure to explain how the liquid's temperature will change.

**ADDING INTEGERS WITH COUNTERS 2**

Follow your teacher's directions to complete problems 1-3.

1.	2.	3.
----	----	----

Compute each sum. Draw using positive symbols (+) and negative symbols (-).

4. $(7) + (-2)$	5. $(5) + (-5)$	6. $(-6) + (6)$
7. $(-4) + (6)$	8. $(1) + (-9)$	9. $(-8) + (3)$

For problems 10-12, use **positive**, **negative**, and **zero** as choices to finish the following sentences. Use all that apply for each.

10. When adding two positive integers, the result will be \_\_\_\_\_.
11. When adding two negative integers, the result will be \_\_\_\_\_.
12. When adding a positive integer and a negative integer, the result will be \_\_\_\_\_, \_\_\_\_\_, or \_\_\_\_\_.
13. Describe how you might add 1 and -2 using Mortimer's magic cubes. Be sure to explain how the liquid's temperature will change.

**INTEGER ADDITION RULES**

<b>Description of counters on your workspace</b>	<b>Drawing</b>	<b>Numerical example</b>	<b>Summarizing Shorthand</b> (pos → positive) (neg → negative)
<b>1. Positive Only</b>			
Place some positives. Then place more positives.		____ + ____	pos + pos is
<b>2. Negative Only</b>			
Place some negatives. Then place more negatives.		____ + ____	neg + neg is
<b>3. Positive And Negative</b>			
a. Place some of each so that the result is positive.		____ + ____	pos + neg is pos when:
b. Place some of each so that the result is negative.		____ + ____	pos + neg is neg when:
c. Place some of each so that the result is zero.		____ + ____	pos + neg is 0 when:

**PRACTICE 2**

Without computing, determine whether each sum is positive (pos), negative (neg), or zero (0).

<b>Positive Example</b> $6 + (-4)$		<b>Negative Example</b> $-6 + (-4)$		<b>Zero Example</b> $-6 + 6$	
1.	$-2 + (-11)$	2.	$7 + (-3)$	3.	$-2 + (-6)$
4.	$9 + 4$	5.	$-6 + 4$	6.	$11 + (-4)$
7.	$-6 + (-1)$	8.	$-5 + 1$	9.	$1 + (-1)$

Compute each sum. Use pictures if desired.

10.	$7 + (-2)$	11.	$-9 + 9$	12.	$-1 + (-3)$
13.	$11 + 12$	14.	$3 + (-8)$	15.	$-5 + 6$
16.	$2 + (-2)$	17.	$-3 + (-6)$	18.	$-13 + 3$

Make each equation true using the given directions.

<b>Directions</b>	<b>Equation</b>
19. Both numbers are positive.	$10 = \underline{\quad} + \underline{\quad}$
20. One positive number and one negative number.	$10 = \underline{\quad} + \underline{\quad}$
21. Both numbers are negative.	$-10 = \underline{\quad} + \underline{\quad}$
22. One positive number and one negative number.	$-10 = \underline{\quad} + \underline{\quad}$

**PRACTICE 3**

1. Compute the following:  $4 + 4 = \underline{\quad}$  and  $-4 + (-4) = \underline{\quad}$ .

How is adding two negative numbers the same as adding two positive numbers?

How is it different?

2. Compute the following:  $5 + (-2) = \underline{\quad}$        $-5 + 2 = \underline{\quad}$

How is computing these related to subtraction?

Compute each sum without using counters or drawing pictures. If not done mentally, show your work.

3. $-10 + (-20)$	4. $-100 + 200$	5. $-80 + 60$
6. $-75 + (-25)$	7. $-100 + 1$	8. $-80 + 120$
9. $-26 + 56$	10. $12 + (-48)$	11. $-53 + (-27)$
12. $57 + (-89)$	13. $(-17) + 68$	14. $-37 + (-56)$

## SUBTRACTING INTEGERS

We will use counters to generalize rules for integer subtraction.

### GETTING STARTED

1. Build and draw a value of  $-2$  in two different ways. For both, use more than 6 counters, but fewer than 12 counters.

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2. How many ways are there to build any given number? \_\_\_\_\_

Compute each sum. Use positive symbols (+) and negative symbols (–) if desired.

3. $-6 + (-3)$	4. $-6 + 3$
5. $6 + (-3)$	6. $2 + (-2)$

Compute each sum without using counters or drawing pictures. Show your work if you don't solve mentally.

7. $60 + (-30)$	9. $27 + (-59)$
8. $-600 + (-300)$	

10. Think about Mortimer's magic cubes. Regardless of the temperature of the liquid,
- a. what happens to it if we take out some cold cubes?
  - b. what happens if instead we take out some hot cubes?

## SUBTRACTING INTEGERS WITH COUNTERS 1

Use these sentence frames to help you think through integer subtraction. **Do not write in these.**

- Begin with a workspace that has a value equal to 0.
- Build \_\_\_\_\_  
positive/negative
- The minus ( - ) means to subtract.
- Subtract \_\_\_\_\_ counter(s). Introduce zero pairs if needed.  
positive/negative
- The result is \_\_\_\_\_ counter(s).  
positive/negative

Follow your teacher's directions to complete problems 1-3.

1.	2.	3.
----	----	----

Compute each difference. Draw using positive symbols (+) and negative symbols (-).

4. $6 - 3$	5. $-6 - (-3)$	6. $-7 - (-1)$
7. $-1 - (-1)$	8. $4 - 4$	9. $-4 - (-4)$

10. Mateo remembers that someone told him “when you subtract, the result is **less than** what you started with.” Look at problems 1-9 above for any instances where “when you subtract, the result is **more than** what you started with,” and draw a star next to those problems.

**SUBTRACTING INTEGERS WITH COUNTERS 2**

Follow your teacher's directions to complete problems 1-4.

1.	2.
3.	4.

Compute each difference. Draw using positive symbols (+) and negative symbols (-).

5. $1 - 3$	6. $-4 - (-5)$
7. $2 - (-4)$	8. $-6 - (3)$
9. $8 - (-4)$	10. $3 - (-3)$
11. $-5 - (5)$	12. $-2 - (-2)$

13. Look at problems 1-12 above for any instances where “when you subtract, the result is more than what you started with,” and draw a star next to those problems. On this page and the previous page, look at all the problems where you put stars. What do you notice about the number that is being subtracted EVERY time?

### SUBTRACTION RULES

Value on your workspace (We will call it $M$ )	Draw a picture to represent this action on $M$ (Do you need zero pairs?)	Equation	Result is greater or less than $M$ ? (circle)
1. $M = 3$ + + +	Subtract 2 + + +	$3 - 2 = \underline{\quad}$	$> M$ $< M$
2. $M = -3$ - - -	Subtract 2 - - -		$> M$ $< M$
3. $M = 3$ + + +	Subtract -2 + + +		$> M$ $< M$
4. $M = -3$ - - -	Subtract -2 - - -		$> M$ $< M$

5. Focus on the action and how it affected  $M$  in the result of the equation. Explain what is different in problems 1 and 2 compared to problems 3 and 4.

Match the equivalent expressions below.

6. $3 - 2$	a. $-3 + 2$
7. $-3 - 2$	b. $3 + (-2)$
8. $3 - (-2)$	c. $3 + 2$
9. $-3 - (-2)$	d. $-3 + (-2)$

**SUBTRACTION RULES**

(Continued)

10. Circle the expression that is equivalent to  $5 - (-7)$ .

$-5 - 7$

$5 - 7$

$5 + 7$

11. Circle the expression that is equivalent to  $5 - 7$ .

$-5 + 7$

$5 + (-7)$

$5 + 7$

Rewrite each subtraction expression as an equivalent addition expression.

12.  $-5 - 7$

13.  $-5 - (-7)$

Match the equivalent expressions below.

14.  $6 - 8$

a.  $6 + 8$

15.  $6 - (-8)$

b.  $-6 + (-8)$

16.  $-6 - 8$

c.  $-6 + 8$

17.  $-6 - (-8)$

d.  $6 + (-8)$

18. Here are two ways to write the **subtraction rule** for any numbers  $m$  and  $n$ :

a.  $m - n = m + \underline{\hspace{2cm}}$

In words,  $m$  minus  $n$  is equal to...

b.  $m - (-n) = a + \underline{\hspace{2cm}}$

In words,  $m$  minus the opposite of  $n$  is equal to...

c. Write a numerical example to illustrate part a above.

d. Write a numerical example to illustrate part b above.

**PRACTICE 4**

Rewrite each subtraction expression as an equivalent addition expression.

1. $17 - 24$	2. $-56 - 18$	3. $23 - (-9)$
4. $-19 - (-44)$	5. $-11 - 37$	6. $-41 - (-15)$

7. Rewrite  $7 + 3$  as an equivalent subtraction expression. \_\_\_\_\_  
Which is easier for you to compute, the addition or subtraction expression?

Compute using any method. Show work as needed.

8. $1 - 8$	9. $-2 - 8$	10. $3 - (-7)$
11. $7 - (-2)$	12. $-5 - (-10)$	13. $9 - 2$
14. $50 - (-25)$	15. $-80 - 20$	16. $40 - 60$
17. $34 - 58$	18. $-83 - 26$	19. $-18 - (-59)$

## REVIEW

### COMPARING ADDITION AND SUBTRACTION

Complete the tables using patterns.

1.

Expression	Sum
5 + ( )	
5 + ( )	
5 + (1)	
5 + (0)	5
5 + (-1)	4
5 + (-2)	
5 + ( )	
5 + ( )	
5 + ( )	
5 + ( )	

2.

Expression	Difference
5 - ( )	
5 - ( )	
5 - (-1)	
5 - (0)	5
5 - (1)	4
5 - (2)	
5 - (3)	
5 - ( )	
5 - ( )	
5 - ( )	

Complete the problems below based on the results in the tables.

3. Under what circumstances are the results **less than 5**?

Adding a \_\_\_\_\_ number or subtracting a \_\_\_\_\_ number.

4. Under what circumstances are the results **greater than 5**?

Adding a \_\_\_\_\_ number or subtracting a \_\_\_\_\_ number.

5. What two expressions have a result of 4? \_\_\_\_\_ and \_\_\_\_\_

6. What two expressions have a result of 8? \_\_\_\_\_ and \_\_\_\_\_

7. Subtracting 6 from a number gives the same result as adding \_\_\_\_\_ to it.

8. Subtracting -2 from a number gives the same result as adding \_\_\_\_\_ to it.

9. Write the related addition expression for each subtraction expression.

a.  $-5 - 1$

b.  $-5 - (-1)$

c.  $0 - (-1)$

## INTEGER BATTLE

You will need:

- 2 or more players
- Integer Cards

Integer Battle is like the classic card game War. It may be played one-on-one or two-on-two.

### Addition version

- Shuffle all the cards and deal out equally to each player/team.
- Both players/teams place two cards from the top of their stack in front of them.
- Each team adds the values on both pairs of cards. The player/team with the greater sum wins, and that player/team collects all four cards in a pile in front of them.
- When a player/team runs out of cards, and there are still collected cards in their pile, they shuffle and reuse those cards like before.
- When a player/team completely runs out of cards, and have none left at all, the other team is declared the winner.

### Subtraction version

- The game is played exactly like the addition version, with one exception. When two cards are placed down, order matters. The second card placed down is subtracted from the first card placed down. Therefore, this version requires that players are careful to note which card is drawn first, and which is drawn second.

1. Play the **addition** version of Integer Battle. Record two winning hands:

$$1^{\text{st}} \quad \underline{\quad} + \underline{\quad} = \underline{\quad} \text{ is greater than } \underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$2^{\text{nd}} \quad \underline{\quad} + \underline{\quad} = \underline{\quad} \text{ is greater than } \underline{\quad} + \underline{\quad} = \underline{\quad}$$

2. Play the **subtraction** version of Integer Battle. Record two winning hands:

$$1^{\text{st}} \quad \underline{\quad} + \underline{\quad} = \underline{\quad} \text{ is greater than } \underline{\quad} + \underline{\quad} = \underline{\quad}$$

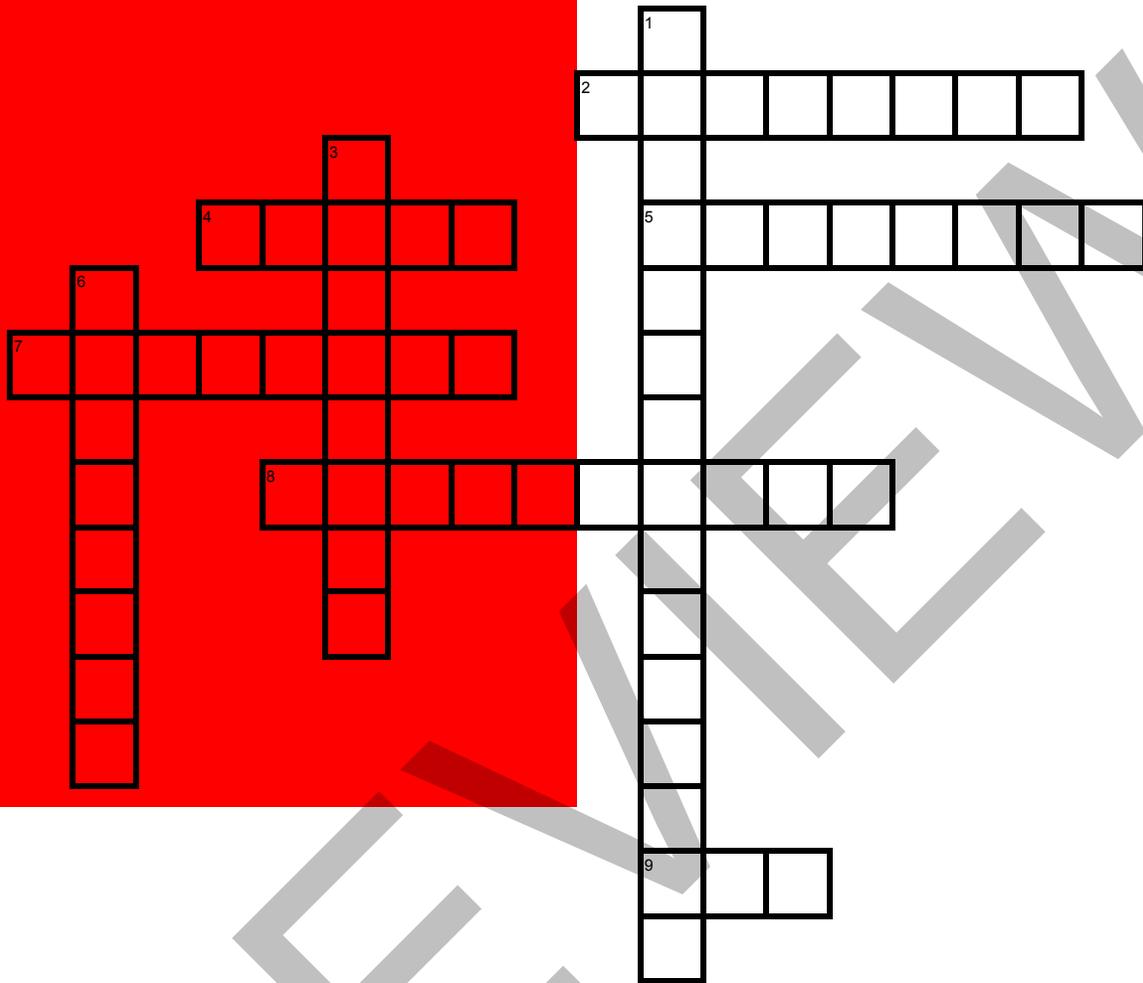
$$2^{\text{nd}} \quad \underline{\quad} + \underline{\quad} = \underline{\quad} \text{ is greater than } \underline{\quad} + \underline{\quad} = \underline{\quad}$$

### **BIG SQUARE PUZZLE: ADDING AND SUBTRACTING INTEGERS**

Your teacher will give you a “big square puzzle” to complete. Assemble the pieces so that each addition or subtraction expression matches its result and tape the solution here.

PREVIEW

## VOCABULARY REVIEW

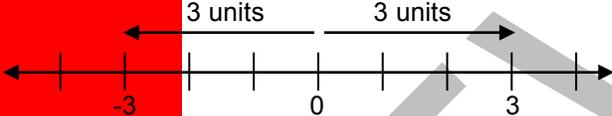
**Across**

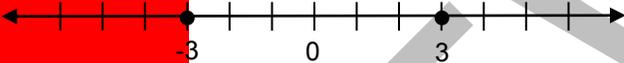
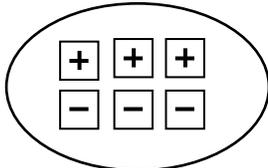
- 2 The \_\_\_\_\_ identity property states that the sum of a number and zero is the number itself.
- 4 \_\_\_\_\_ numbers are the numbers 0, 1, 2, 3, ... .
- 5 \_\_\_\_\_ are the numbers 0, 1, 2, 3, ... together with -1, -2, -3, ... .
- 7 A \_\_\_\_\_ number is less than zero.
- 8 A \_\_\_\_\_ is the result of subtraction.
- 9 A \_\_\_\_\_ is the result of addition.

**Down**

- 1 The \_\_\_\_\_ \_\_\_\_\_ property states that the sum of a number and its opposite is zero.
- 3 A \_\_\_\_\_ number is greater than zero.
- 6 The combination of one positive counter and one negative counter is a \_\_\_\_\_ \_\_\_\_\_.

## DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
absolute value	<p>The <u>absolute value</u> <math> x </math> of a number <math>x</math> is the distance from <math>x</math> to 0 on the number line.</p> <p><math> 3  = 3</math> and <math> -3  = 3</math>, because both 3 and -3 are 3 units from 0 on the number line.</p> 
addend	<p>In an addition problem, an <u>addend</u> is a number to be added. See <u>sum</u>.</p> $\begin{array}{rcccc} 7 & + & 5 & = & 12 \\ \text{addend} & & \text{addend} & & \text{sum} \end{array}$
additive identity property	<p>The <u>additive identity property</u> states that <math>a + 0 = 0 + a = a</math> for any number <math>a</math>. In other words, the sum of a number and 0 is the number.</p> <p>We say that 0 is an <u>additive identity</u>. The additive identity property is sometimes called the <u>addition property of zero</u>.</p> $3 + 0 = 3, 0 + 7 = 7, -5 + 0 = -5 = 0 + (-5)$
additive inverse	<p>The <u>additive inverse</u> of <math>a</math> is the number <math>b</math> such that <math>a + b = b + a = 0</math>. The additive inverse of <math>a</math> is denoted by <math>-a</math>.</p> <p>-4 is the additive inverse of 4.</p>
additive inverse property	<p>The <u>additive inverse property</u> states that <math>a + (-a) = 0</math> for any number <math>a</math>. In other words, the sum of a number and its opposite is 0. The number <math>-a</math> is the additive inverse of <math>a</math>.</p> $3 + (-3) = 0, -5 + 5 = 0$
difference	<p>In a subtraction problem, the <u>difference</u> is the result of subtraction. The <u>minuend</u> is the number from which another number is being subtracted, and the <u>subtrahend</u> is the number that is being subtracted.</p> $\begin{array}{rcccc} 12 & - & 4 & = & 8 \\ \text{minuend} & & \text{subtrahend} & & \text{difference} \end{array}$
integers	<p>The <u>integers</u> are the whole numbers and their opposites. They are the numbers 0, 1, 2, 3, ... and -1, -2, -3, ... .</p>
minuend	<p>In a subtraction problem, the <u>minuend</u> is the number from which another is subtracted. See <u>difference</u>.</p>

Word or Phrase	Definition
negative number	<p>A <u>negative number</u> is a number that is less than zero, written <math>a &lt; 0</math>. The negative numbers are the numbers to the left of 0 on a horizontal number line, or below zero on a vertical number line.</p> <p>The numbers -2, -4.76, and <math>-\frac{1}{4}</math> are negative.</p> <p>The numbers 2 and 5.3 are not negative. They are positive. The number 0 is neither negative nor positive.</p>
opposite of a number	<p>The <u>opposite of a number</u> <math>n</math>, written <math>-n</math>, is its additive inverse. Algebraically, the sum of a number and its opposite is zero. Geometrically, the opposite of a number is the number on the other side of zero at the same distance from zero.</p>  <p>The opposite of 3 is -3, because <math>3 + (-3) = -3 + 3 = 0</math>.          The opposite of -3 is <math>-(-3) = 3</math>.          Thus, the opposite of a number does not have to be negative.</p>
positive number	<p>A <u>positive number</u> is a number that is greater than zero, written <math>a &gt; 0</math>. The positive numbers are the numbers to the right of 0 on a number line, or above zero on a vertical number line.</p> <p>The numbers 3, 2.6, and <math>\frac{3}{7}</math> are positive.</p> <p>The numbers -3, -2.6, <math>-\frac{3}{7}</math>, and 0 are NOT positive.</p>
subtrahend	<p>In a subtraction problem, the <u>subtrahend</u> is the number that is being subtracted from another. See <u>difference</u>.</p> <p>In <math>12 - 4 = 8</math>, the subtrahend is 4.</p>
sum	<p>A <u>sum</u> is the result of addition. In an addition problem, the numbers to be added are <u>addends</u>.</p> $\begin{array}{ccccccc} 7 & + & 5 & = & 12 \\ \text{addend} & & \text{addend} & & \text{sum} \end{array}$
whole numbers	<p>The <u>whole numbers</u> are the natural numbers together with 0. They are the numbers 0, 1, 2, 3, ... .</p>
zero pair	<p>In the counter model, a positive and a negative counter together form a <u>zero pair</u>.</p> <p>Let <math>\boxed{+}</math> represent a positive counter and</p> <p>let <math>\boxed{-}</math> represent a negative counter.</p> <p>Then the figure to the right is an example of a collection of (three) zero pairs.</p> 

**A Counter Model**

This counter model is used to model integers.

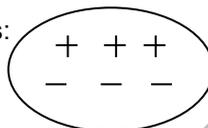
Let + represent a positive counter with a value of positive 1  
 Let - represent a negative counter with a value of negative 1.

A zero pair is a pair with one positive counter and one negative counter.  
 Both representations below have a value of zero.

one zero pair:



three zero pairs:



Below are some counter diagrams that represent the given integers:

	+4	-2	0
<b>Simplest representation:</b>	+ + + +	- -	(no counters)
<b>Other representations:</b>	+ + + + + -	- - + + - -	+ + - -
	+ + + + + + + - - -	- - - - - + + + + +	+ + + + + - - - - -

**Mr. Mortimer's Magic Hot and Cold Cubes**

Mr. Mortimer discovered an amazing way to control the temperature of liquid. He invented magic hot and cold cubes to change the temperature of the liquid. These magic cubes never melt or change in any way.

Hot Cubes:

- If you add 1 hot cube to a liquid, the liquid heats up by 1 degree.
- If you remove 1 hot cube from the liquid, the liquid cools down by 1 degree.

Cold Cubes:

- If you add 1 cold cube to the liquid, the liquid cools down by 1 degree.
- If you remove 1 cold cube from the liquid, the liquid heats up by 1 degree.

How this temperature change model works			For 1 cube
Hot Cubes Positive (+)	Put in Heat →	Hotter	add (+1) → $+(+1) = +1$
	Remove Heat →	Colder	subtract (+1) → $-(+1) = -1$
Cold Cubes Negative (-)	Put in Cold →	Colder	add (-1) → $+(-1) = -1$
	Remove Cold →	Hotter	subtract (-1) → $-(-1) = +1$

Here are a few examples to show temperature change using magic hot and cold cubes.

	Simplest ways:		Other Ways:	
<b>+4 degrees</b>	Put in 4 hot cubes $+(+4) = 4$	Remove 4 cold cubes $-(-4) = 4$	Put in 6 hot cubes and put in 2 cold cubes $+(+6) + (-2) = 4$	Remove 6 cold cubes and remove 2 hot cubes $-(-6) - (+2) = 4$
<b>-2 degrees</b>	Remove 2 hot cubes $-(+2) = -2$	Put in 2 cold cubes $+(-2) = -2$	Remove 3 hot cubes and remove 1 cold cube $-(+3) - (-1) = -2$	Put in 3 cold cubes and put in 1 hot cube $+(-3) + (+1) = -2$
<b>0 degrees</b>	Do nothing 0		Put in 4 hot cubes and put in 4 cold cubes $+(+4) + (-4) = 0$	Remove 3 hot cubes and remove 3 cold cubes $-(+3) - (-3) = 0$

Representing the Additive Inverse	
<p>The minus sign may be used to show additive inverses. The identity <math>a + (-a) = 0</math> means that <math>-a</math> is the additive inverse of <math>a</math>. It is what we add to <math>a</math> to get 0.</p>	<p style="text-align: right;">Example: If <math>a = -3</math>, then <math>-a = 3</math></p> <p>The statement, "If <math>a</math> is equal to minus 3, then minus <math>a</math> is equal to 3" can be read:</p> <ul style="list-style-type: none"> <li>If <math>a</math> is equal to the opposite of 3, then the opposite of <math>a</math> is equal to 3. When we add <math>-3</math> and 3, the result is 0.</li> </ul> <div style="text-align: center; margin-top: 10px;"> </div>

Counter Addition Sentence Frames
<ul style="list-style-type: none"> <li>Begin with a workspace that has a value equal to 0.</li> <li>Build _____ positive/negative</li> <li>The plus (+) means to add.</li> <li>Add _____ counter(s). positive/negative</li> <li>The result is _____ counter(s). positive/negative</li> </ul>

Integer Addition Using Counters		
<p style="text-align: center;"><math>-3 + (-5) = -8</math></p> <p style="text-align: center;">                     — — —                      — — — — —                 </p> <ul style="list-style-type: none"> <li>Start with a work space equal to zero.</li> <li>Build negative 3.</li> <li>The (+) means to add.</li> <li>Add 5 negative counters.</li> <li>The result is 8 negative counters.</li> </ul>	<p style="text-align: center;"><math>-3 + 5 = 2</math></p> <p style="text-align: center;">                     — — —                      + + + + +                 </p> <ul style="list-style-type: none"> <li>Start with a work space equal to zero.</li> <li>Build negative 3.</li> <li>The (+) means to add.</li> <li>Add 5 positive counters.</li> <li>The result is 2 positive counters.</li> </ul>	<p style="text-align: center;"><math>3 + (-5) = -2</math></p> <p style="text-align: center;">                     + + +                      — — — — —                 </p> <ul style="list-style-type: none"> <li>Start with a work space equal to zero.</li> <li>Build positive 3.</li> <li>The (+) means to add.</li> <li>Add 5 negative counters.</li> <li>The result is 2 negative counters.</li> </ul>

**Rules for Addition of Integers**

**Rule 1:** When the addends have the same sign, add the absolute values. Use the original sign in the answer.

**Rule 2:** When the addends have different signs, subtract the absolute values. Use the sign of the addend with the greatest absolute value in the answer.

**Counter Subtraction Sentence Frames**

- Begin with a workspace that has a value equal to 0.
- Build \_\_\_\_\_  
positive/negative
- The minus ( - ) means to subtract.
- Subtract \_\_\_\_\_ counter(s). Introduce zero pairs if needed.  
positive/negative
- The result is \_\_\_\_\_ counter(s).  
positive/negative

**Integer Subtraction Using Counters**

$-5 - (-3) = -2$



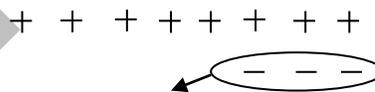
- Start with a work space equal to zero.
- Build negative 5.
- The (-) means to subtract.
- I do not need zero pairs.
- Subtract 3 negative counters.
- The result is 2 negative counters.

$-3 - (-5) = 2$



- Start with a work space equal to zero.
- Build negative 3.
- The (-) means to subtract.
- Subtract 5 negative counters. I need zero pairs to do this.
- The result is 2 positive counters.

$5 - (-3) = 8$



- Start with a work space equal to zero.
- Build positive 5.
- The (-) means to subtract.
- Subtract 3 negative counters. I need zero pairs to do this.
- The result is 8 positive counters.

**Rule for Subtraction of Integers**

**Rule:** In symbols,  $a - b = a + (-b)$  and  $a - (-b) = a + b$ .

In words, subtracting a quantity gives the same result as adding its opposite.

Example:  $6 - 4 = 6 + (-4) = 2$

